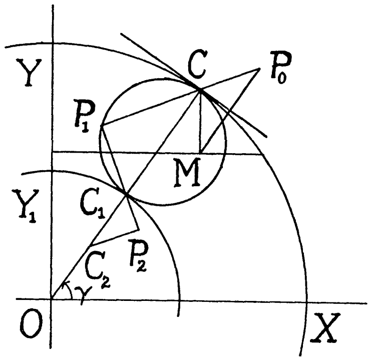
II. SOLUTION BY OTTO DUNKEL, Washington University.

The solution of this problem, like that of 2819 (*1921*, 190) is simplified by the geometry of the curves. These may be constructed as follows: A unit circle is drawn with the origin as center and a chord parallel to the x-axis at a units above it. A radius OC is drawn with the inclination γ to the x-axis and the point C is projected upon the chord in the point M. Then the point P_0 which is symmetrical to M with respect to the tangent at C is a point on the given curve, P_0C is a normal, and the envelope of this normal, the first evolute of the present problem, is the caustic of the unit circle produced by vertical rays. If P_1 is the point of contact with the caustic, $CP_1 = (\sin\gamma)/2$ (formula (1), *1920*, 225, ω of the formula being equal to $\pi/2 - \gamma$ and R to 1), and this point is obtained by projecting the middle point C_1 of OC upon P_0C. P_1P_0 has the inclination $2\gamma - \pi/2$ and length $\rho_0 = (3/2)\sin\gamma - a$. Angle P_1CC_1 = angle YOC_1; therefore the arc P_1C_1 on the circle of diameter C_1C is equal to the arc Y_1C_1, where Y_1 is the point where the y-axis is cut by the circle with center O and radius $OC_1 = 1/2$. It follows that the locus of P_1 is the curve traced by this point when the former circle rolls on the latter circle; it is an epicycloid of two cusps, one at Y_1 and the other at the diametrically opposite point. This is our first evolute and P_1C_1 is its normal.



If $\rho_1 = P_1P_2$ is the radius of curvature of the first evolute, then $\rho_1 = (1/2)d\rho_0/d\gamma = 3(\cos\gamma)/4$, $C_1P_2 = (1/2)\,P_1C_1$, and P_2 is the projection upon this line of C_2 the middle point of OC_1. Now we prove as above for P_1, that the locus of P_2, our second evolute, is a two-cusped epicycloid, this time traced by rolling the circle of diameter C_2C_1 upon the fixed circle of radius OC_2, and having its cusps at the point where the latter circle cuts the x-axis and at the diametrically opposite point.

The same reasoning may be repeated again and again, giving us for the nth evolute an epicycloid with cusps on the x-axis when n is even and on the y-axis when n is odd. The equations in the former case are $x_n = (1/2^{n+1})(3\cos\gamma - \cos 3\gamma)$, $y_n = (1/2^{n+1})(3\sin\gamma - \sin 3\gamma)$, while the minus signs are changed to plus signs for the latter case.

The length of an arc of any evolute after the first measured from the nearest cusp of the preceding evolute is equal to the radius of curvature of the preceding evolute. Now $\rho_{n-1} = 3(\sin\gamma)/2^n$ or $3(\cos\gamma)/2^n$, neglecting signs; hence the complete length of the nth evolute can be obtained from one or the other of these expressions by putting the sine or cosine equal to 1 and multiplying by 4. That is, it is $3/2^{n-2}$ (it is 6 for $n = 1$).

For an evolute whose cusps lie on the y-axis the element of area generated by that portion of the radius of curvature which lies outside of the corresponding fixed circle is equal to twice the element xdy for the circle. For the first evolute, for example, it is $(1/2)\cos^2\gamma d\gamma$. A similar relation, the axes being interchanged, holds for an evolute whose cusps lie on the x-axis. Therefore the area of any evolute of the system is 3 times the area of the corresponding circle; namely, for the nth evolute it is $3\pi/2^{2n}$.

Since the lengths and areas form geometrical progressions it is easy to find their sums.

NOTES AND NEWS.

It is hoped that readers of the MONTHLY will coöperate in contributing to the general interest of this department by sending items to H. P. MANNING, Brown University, Providence, R. I.

CHARLES LEONARD BOUTON died at Cambridge, Mass., February 20, 1922. He was born at St. Louis, April 25, 1869. He graduated at the Washington University with the degree of M.Sc. in 1891, took the degree of A.M. at

Harvard in 1896, and the degree of Ph.D. at Leipzig in 1898. He was instructor at Smith Academy, St. Louis, 1891–1894, and at Washington University, 1893–1894. He went to Harvard as instructor in 1898, became assistant professor in 1904 and associate professor in 1914. He was one of the editors of the *Bulletin of the American Mathematical Society*, 1900–1902, and of the *Transactions*, 1902–1910. He contributed several articles to the *Annals of Mathematics* and to other periodicals, the most important being "Invariants of the general linear differential equation and their relation to the theory of continuous groups," *American Journal of Mathematics*, volume 21, 1899, pages 25–84, and "Examples of the construction of Riemann's surfaces for the inverse of rational functions, by the method of conformal representation," *Annals of Mathematics*, volume 12, 1898–1899, pages 1–26. The former was his dissertation under Lie at Leipzig.

GEORGE BRUCE HALSTED died in New York, March 19, 1922. Professor Halsted was born in Newark, N. J., November 25, 1853. He received the degree of A.B. from Princeton in 1875, and in 1879 the degree of Ph.D. from Johns Hopkins, where he was fellow, 1876–1878. He was instructor at Princeton, 1878–1881, and professor at the University of Texas, 1884–1903, at St. Johns College, Md., 1903, at Kenyon College, Ohio, 1903–1906, and at the Colorado State Teachers College, 1906–1912.

His first book was *Mensuration*, Boston, 1881. In New York, appeared *Elements of Geometry*, 1885; *Elementary Synthetic Geometry*, 1892; "Projective Geometry," Chapter II in *Higher Mathematics*, edited by Mansfield Merriman and R. S. Woodward, 2d ed., 1898, published separately as No. 2 of *Mathematical Monographs*, 1906; *Rational Geometry*, 1904, revised 1907 (translated into French by P. Barbarin, Paris, 1911).

His most important work was the translation of writings on non-Euclidean geometry. Lobachevski's *Researches on the Theory of Parallels* and Bolyai's *Science Absolute of Space* were first published at Austin, Texas, in 1891, as parts of the "Neomonic Series." They are now published in Chicago. His translation of Saccheri, *Euclides Vindicatus*, as *Euclid freed from every Fleck*, first appeared (all but the last thirteen pages) in this MONTHLY, *1894–1898*. He also translated some of Poincaré's writings on the foundations of science; namely, *Science and Hypothesis*, *The Value of Science*, and *Science and Method*, published in a single volume in 1913, New York and Garrison, N. Y.

Professor Halsted wrote numerous articles, biographical sketches, criticisms, etc., which are scattered through this MONTHLY, *Science* and other periodicals. Sommerville gives a list of ninety titles in his *Bibliography of Non-Euclidean Geometry* (London, 1911).

The honorary degree of doctor of science has been conferred on Sir THOMAS MUIR by the University of Cape Town, in recognition of his researches in mathematics and the history of mathematics.

Professor E. I. FREDHOLM, of the University of Stockholm, has been elected correspondent of the Paris Academy of Sciences in the section of geometry, as successor to the late Professor H. A. Schwarz.

A colloquium on the fundamental concepts of electrodynamics and of the electron theory of matter was held at the University of Wisconsin on March 30, 31, and April 1, 1922. The particular occasion for this meeting was the presence of Professor H. A. LORENTZ, the founder of the electron theory. The majority of those present were from the universities and colleges of the middle west, although both the Atlantic and Pacific coasts were represented. During the week preceding the colloquium proper, Dr. Lorentz gave four lectures on the general subject of light and the constitution of matter. These lectures, attended by a large and enthusiastic group of students and physicists, began with the basic concepts of the electromagnetic field, and traced briefly the developments which have led to the modern viewpoint. Professor Lorentz considered the successes and logical difficulties of the Bohr-atom theory, as extended by Sommerfeld and others, and discussed at some length the Michelson-Morley experiment and restricted relativity. In the last lecture a quantum-theory explanation of the Zeemann effect was given, to replace the older theory, based on classical electrodynamics.

During the colloquium itself the following lectures were given: "The experimental basis for the laws of electrodynamic action" by W. F. G. SWANN; "Deduction of the laws of electrodynamics from the relativity principle" by LEIGH PAGE; "Analytic formulation of electromagnetic theory through the field concept" by MAX MASON; "The structure of the electron" by D. L. WEBSTER; "The rotating earth as a reference system for light propagation" by L. SILBERSTEIN; "Application of statistical mechanics to electron theory" by A. C. LUNN; "Scattering of light and resonance radiation in relation to optical theories" by R. W. WOOD; "Thermal radiation—A discussion of recent experimental results" by C. E. MENDENHALL; "Electron theory of metals, volume phenomena" by P. W. BRIDGEMAN; "Electron theory of metals, surface phenomena" by P. T. COMPTON. At the conclusion of each paper the discussion was opened by Dr. Lorentz. The searching keenness, kindly interest, and revealing inspiration of his criticism will undoubtedly stimulate the scientific activities of all those who were present.

Published October 15, 1922.